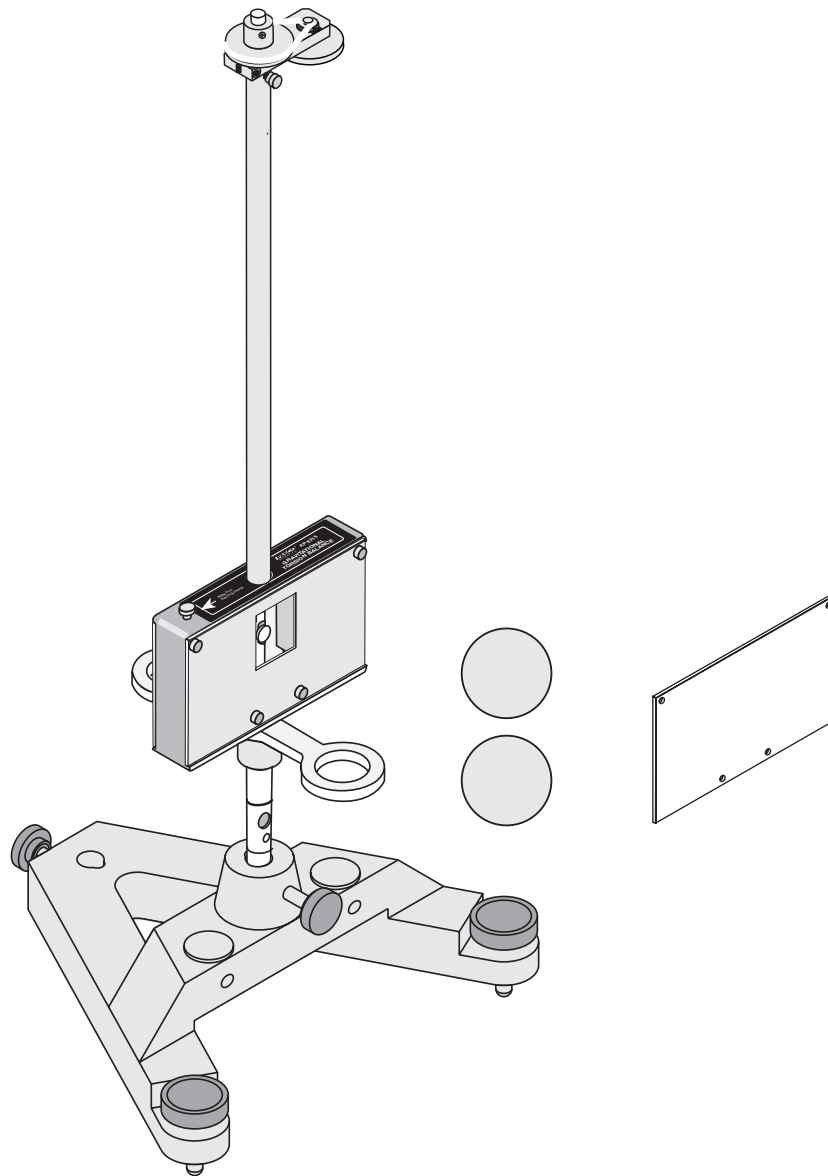


**Instruction Manual and  
Experiment Guide for the  
PASCO scientific  
Model AP-8215**

012-06802B

11/98

# **GRAVITATIONAL TORSION BALANCE**



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***Note:*** To avoid breaking the torsion ribbon, the locking mechanism must be fully raised on both sides during moving or transporting the Gravitational Torsion Balance.



***Note:*** Save the packing material from the interior of the chamber, and re-install this material during moving or transporting the Gravitational Torsion Balance.



The exclamation point within an equilateral triangle is intended to alert the user of the presence of important operating and maintenance (servicing) instructions in the literature accompanying the device.

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- ② Make certain there are at least two inches of packing material between any point on the apparatus and the inside walls of the carton.
- ③ Make certain that the packing material cannot shift in the box or become compressed, allowing the instrument come in contact with the packing carton.

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## Introduction

The PASCO scientific AP-8215 Gravitational Torsion Balance reprises one of the great experiments in the history of physics—the measurement of the gravitational constant, as performed by Henry Cavendish in 1798.

The Gravitational Torsion Balance consists of two 38.3 gram masses suspended from a highly sensitive torsion ribbon and two 1.5 kilogram masses that can be positioned as required. The Gravitational Torsion Balance is oriented so the force of gravity between the small balls and the earth is negated (the pendulum is nearly perfectly aligned vertically and horizontally). The large masses are brought near the smaller masses, and the gravitational force between the large and small masses is measured by observing the twist of the torsion ribbon.

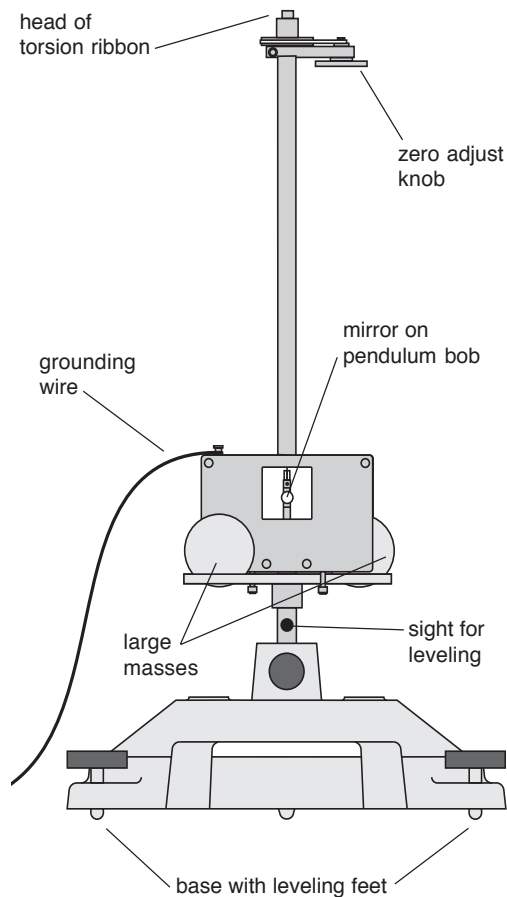
An optical lever, produced by a laser light source and a mirror affixed to the torsion pendulum, is used to accurately measure the small twist of the ribbon. Three methods of measurement are possible: the final deflection method, the equilibrium method, and the acceleration method.

### A Little Background

The gravitational attraction of all objects toward the Earth is obvious. The gravitational attraction of every object to every other object, however, is anything but obvious. Despite the lack of direct evidence for any such attraction between everyday objects, Isaac Newton was able to deduce his law of universal gravitation.

However, in Newton's time, every measurable example of this gravitational force included the Earth as one of the masses. It was therefore impossible to measure the constant,  $G$ , without first knowing the mass of the Earth (or vice versa).

The answer to this problem came from Henry Cavendish in 1798, when he performed experiments with a torsion balance, measuring the gravitational attraction between relatively small objects in the laboratory. The value he determined for  $G$  allowed the mass and density of the Earth to be determined. Cavendish's experiment was so well constructed that it was a hundred years before more accurate measurements were made.



**Figure 1**

Assembled Gravitational Torsion Balance, ready to begin Henry Cavendish's classic experiment to determine the gravitational constant

**Newton's law of universal gravitation:**

$$F = G \frac{m_1 m_2}{r^2}$$

where  $m_1$  and  $m_2$  are the masses of the objects,  $r$  is the distance between them, and

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

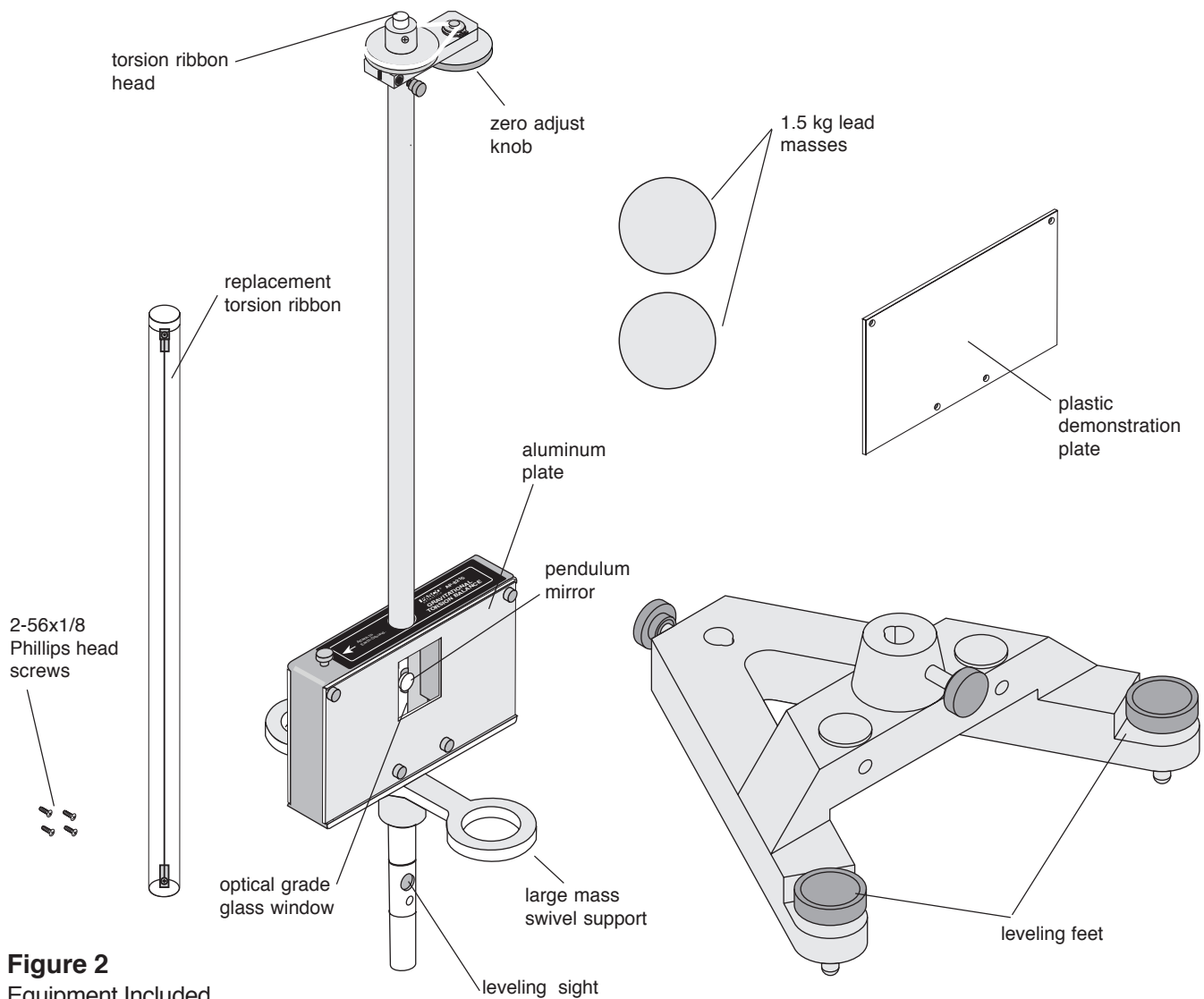
# Equipment

## Included:

- Gravitational Torsion Balance
- support base with leveling feet
- 1.5 kg lead balls (2)
- plastic plate
- replacement torsion ribbon  
(part no. 004-06788)
- 2-56 x 1/8 Phillips head screws (4)
- Phillips screwdriver (not shown)

## Additional Required:

- laser light source (such as the PASCO OS-9171 He-Ne Laser)
- meter stick



**Figure 2**  
Equipment Included

## Equipment Parameters

- Small lead balls
  - Mass:  $38.3 \text{ g} \pm 0.2 \text{ g}$  ( $m_2$ )
  - Radius: 9.53 mm
  - Distance from ball center to torsion axis:  $d = 50.0 \text{ mm}$
- Large lead balls
  - Mass:  $1500 \text{ g} \pm 10 \text{ g}$  ( $m_1$ )
  - Radius: 31.9 mm
- Distance from the center of mass of the large ball to the center of mass of the small ball when the large ball is against the aluminum plate and the small ball is in the center position within the case:  $b = 46.5 \text{ mm}$  (Tolerances will vary depending on the accuracy of the horizontal alignment of the pendulum.)
- Distance from the surface of the mirror to the outer surface of the glass window: 11.4 mm
- Torsion Ribbon Material: Beryllium Copper
  - Length: approx. 260 mm
  - Cross-section: .017 x .150 mm

## Equipment Setup

### Initial Setup

1. Place the support base on a flat, stable table that is located such that the Gravitational Torsion Balance will be at least 5 meters away from a wall or screen.

**Note:** For best results, use a very sturdy table, such as an optics table.

2. Carefully remove the Gravitational Torsion Balance from the box, and secure it in the base.
3. Remove the front plate by removing the thumbscrews (Figure 3), and carefully remove the packing foam from the pendulum chamber.

**Note:** Save the packing foam, and reinstall it each time the Gravitational Torsion Balance is transported.

4. Fasten the clear plastic plate to the case with the thumbscrews.



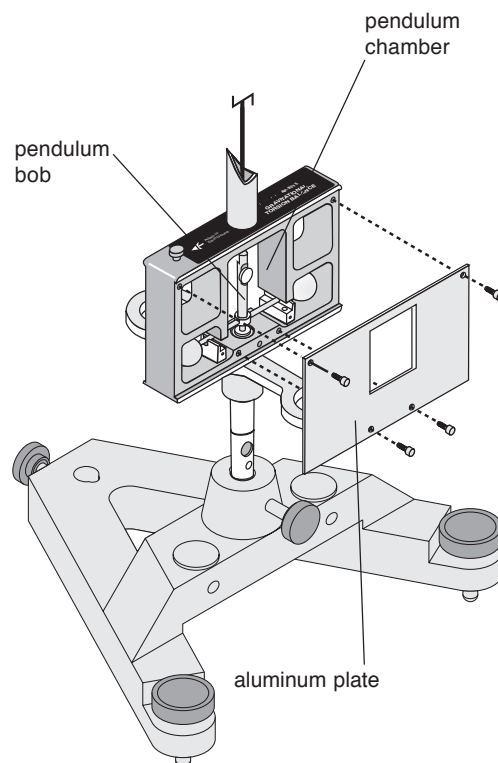
► **Do not touch the mirror on the pendulum.**

## IMPORTANT NOTES

► **The Gravitational Torsion Balance is a delicate instrument. We recommend that you set it up in a relatively secure area where it is safe from accidents and from those who don't fully appreciate delicate instruments.**

► **The first time you set up the torsion balance, do so in a place where you can leave it for at least one day before attempting measurements, allowing time for the slight elongation of the torsion band that will occur initially.**

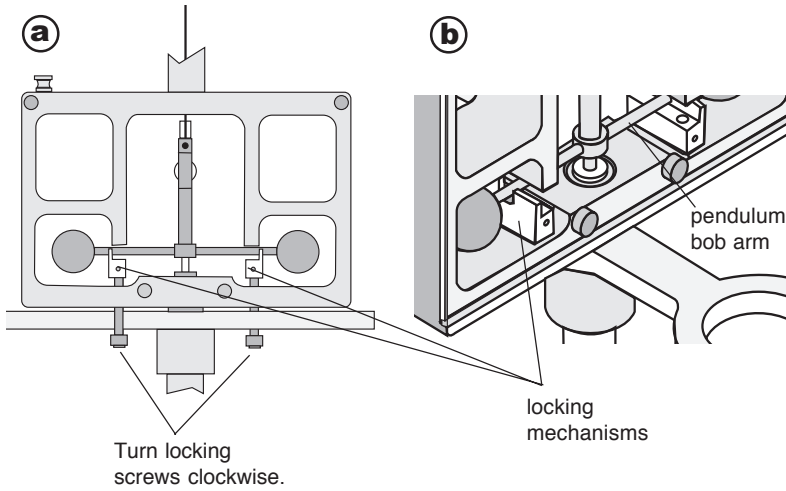
► **Keep the pendulum bob secured in the locking mechanisms at all times, except while setting up and conducting experiments.**



**Figure 3**  
Removing a plate from the chamber box

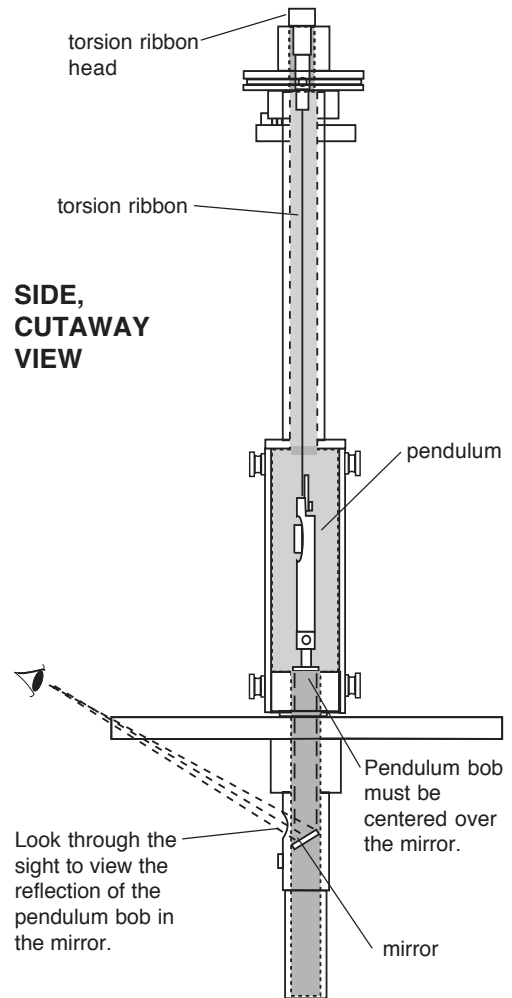
## Leveling the Gravitational Torsion Balance

1. Release the pendulum from the locking mechanism by unscrewing the locking screws on the case, lowering the locking mechanisms to their lowest positions (Figure 4).



**Figure 4**  
Lowering the locking mechanism to release the pendulum bob arms

2. Adjust the feet of the base until the pendulum is centered in the leveling sight (Figure 5). (The base of the pendulum will appear as a dark circle surrounded by a ring of light).
3. Orient the Gravitational Torsion Balance so the mirror on the pendulum bob faces a screen or wall that is at least 5 meters away.



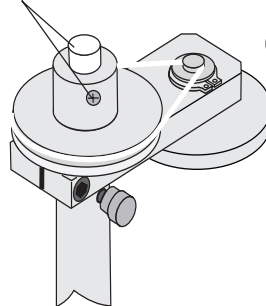
**Figure 5**  
Using the leveling sight to level the Gravitational Torsion Balance.

## Vertical Adjustment of the Pendulum

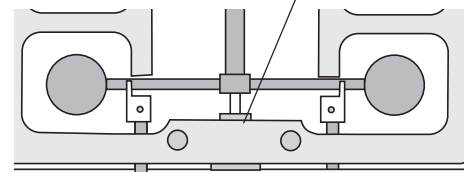
The base of the pendulum should be flush with the floor of the pendulum chamber. If it is not, adjust the height of the pendulum:

1. Grasp the torsion ribbon head and loosen the Phillips retaining screw (Figure 6a).
2. Adjust the height of the pendulum by moving the torsion ribbon head up or down so the base of the pendulum is flush with the floor of the pendulum chamber (Figure 6b).
3. Tighten the retaining (Phillips head) screw.

Grasp the torsion ribbon head and loosen the Phillips screw.



The bottom of the pendulum bob should be flush with the floor of the chamber.



**Figure 6**  
Adjusting the height of the pendulum bob



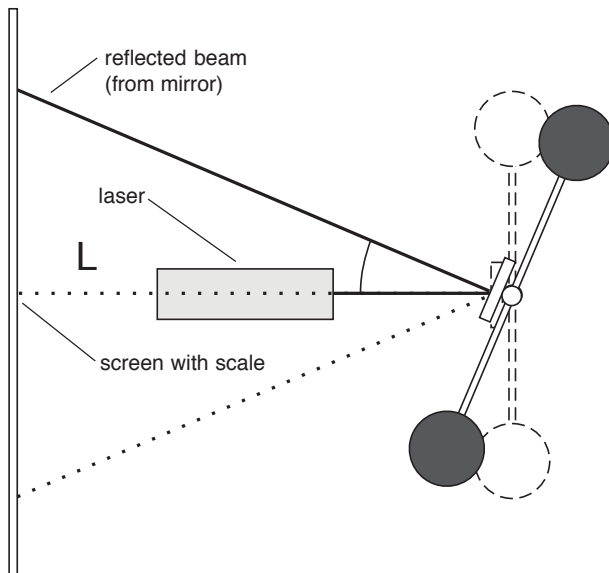
**Note:** Vertical adjustment is only necessary at initial setup and when you change the torsion ribbon or if someone has loosened the retaining screw by mistake; it is not normally done during each experimental setup.

### Rotational Alignment of the Pendulum Bob Arms (Zeroing)

The pendulum bob arms must be centered rotationally in the case — that is, equidistant from each side of the case (Figure 7). To adjust them:

1. Mount a metric scale on the wall or other projection surface that is at least 5 meters away from the mirror of the pendulum.
2. Replace the plastic cover with the aluminum cover.
3. Set up the laser so it will reflect from the mirror to the projection surface where you will take your measurements (approximately 5 meters from the mirror). You will need to point the laser so that it is tilted upward toward the mirror and so the reflected beam projects onto the projection surface (Figure 8). There will also be a fainter beam projected off the surface of the glass window.

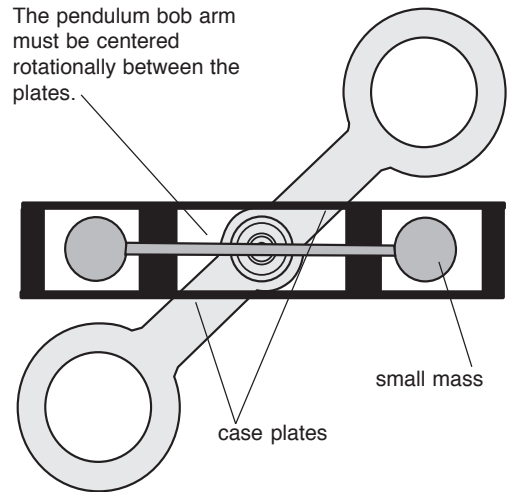
#### TOP VIEW



**Figure 8**  
Setting up the optical lever

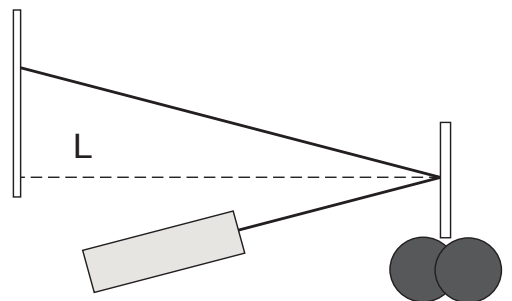
#### TOP, CUTAWAY VIEW

The pendulum bob arm must be centered rotationally between the plates.



**Figure 7**  
Aligning the pendulum bob rotationally

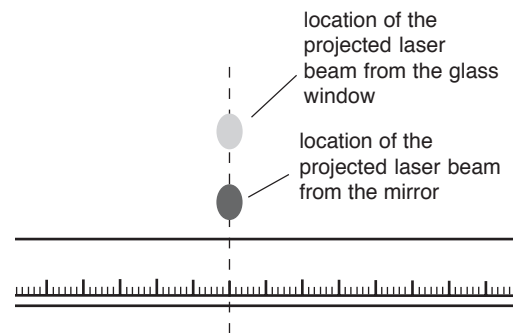
#### SIDE VIEW



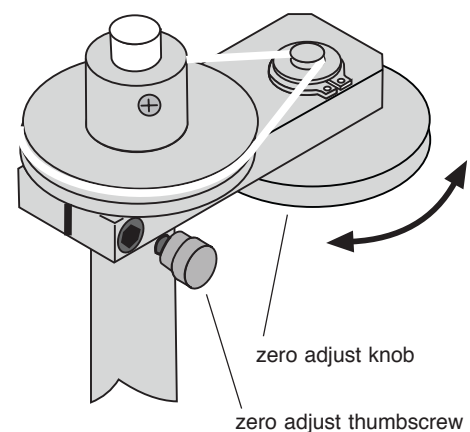
3. Rotationally align the case by rotating it until the laser beam projected from the glass window is centered on the metric scale (Figure 9).
4. Rotationally align the pendulum arm:
  - a. Raise the locking mechanisms by turning the locking screws until both of the locking mechanisms barely touch the pendulum arm. Maintain this position for a few moments until the oscillating energy of the pendulum is dampened.
  - b. Carefully lower the locking mechanisms slightly so the pendulum can swing freely. If necessary, repeat the dampening exercise to calm any wild oscillations of the pendulum bob.
  - c. Observe the laser beam reflected from the mirror. In the optimally aligned system, the equilibrium point of the oscillations of the beam reflected from the mirror will be vertically aligned below the beam reflected from the glass surface of the case (Figure 9).
  - d. If the spots on the projection surface (the laser beam reflections) are not aligned vertically, loosen the zero adjust thumbscrew, turn the zero adjust knob slightly to refine the rotational alignment of the pendulum bob arms (Figure 10), and wait until the movement of the pendulum stops or nearly stops.
  - e. Repeat steps 4a – 4c as necessary until the spots are aligned vertically on the projection surface.
5. When the rotational alignment is complete, carefully tighten the zero adjust thumbscrew, being careful to avoid jarring the system.

#### Hints for speedier rotational alignments:

- Dampen any wild oscillations of the pendulum bob with the locking mechanisms, as described;
- Adjust the rotational alignment of the pendulum bob using small, smooth adjustments of the zero adjust knob;
- Exercise patience and finesse in your movements.



**Figure 9**  
Ideal rotational alignment (zeroing) of the pendulum



**Figure 10**  
Refining the rotational alignment of the pendulum bob

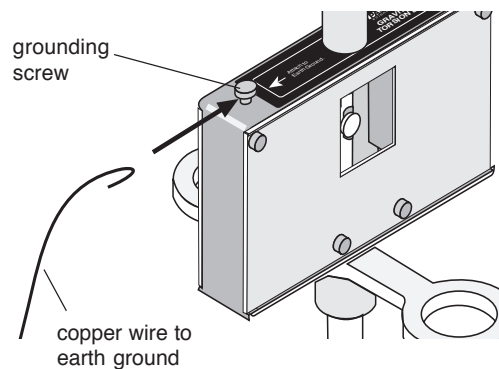
## Setting up for the Experiment

1. Take an accurate measurement of the distance from the mirror to the zero point on the scale on the projection surface ( $L$ ) (Figure 8). (The distance from the mirror surface to the outside of the glass window is 11.4 mm.)

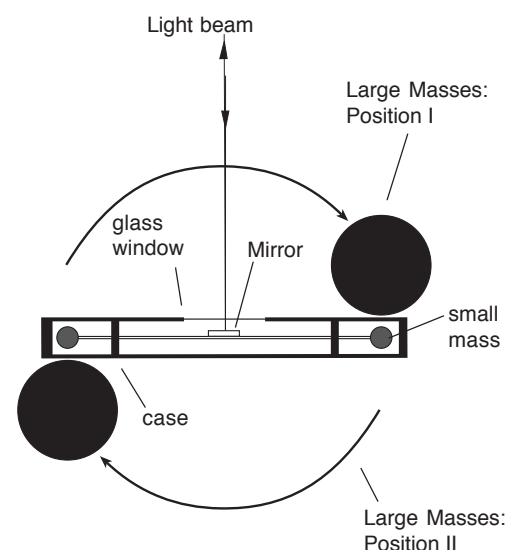
**Note:** Avoid jarring the apparatus during this setup procedure.

2. Attach copper wire to the grounding screw (Figure 11), and ground it to the earth.
3. Place the large lead masses on the support arm, and rotate the arm to Position I (Figure 12), taking care to avoid bumping the case with the masses.
4. Allow the pendulum to come to resting equilibrium.
5. You are now ready to make a measurement using one of three methods: the final deflection method, the equilibrium method, or the acceleration method.

**Note:** The pendulum may require several hours to reach resting equilibrium. To shorten the time required, dampen the oscillation of the pendulum by smoothly raising the locking mechanisms up (by turning the locking screws) until they just touch the crossbar, holding for several seconds until the oscillations are dampened, and then carefully lowering the locking mechanisms slightly.



**Figure 11**  
Attaching the grounding strap to the grounding screw



**Figure 12**  
Moving the large masses into Position I

# Measuring the Gravitational Constant

## Overview of the Experiment

The gravitational attraction between a 15 gram mass and a 1.5 kg mass when their centers are separated by a distance of approximately 46.5 mm (a situation similar to that of the Gravitational Torsion Balance) is about  $7 \times 10^{-10}$  newtons. If this doesn't seem like a small quantity to measure, consider that the weight of the small mass is more than two hundred million times this amount.

The enormous strength of the Earth's attraction for the small masses, in comparison with their attraction for the large masses, is what originally made the measurement of the gravitational constant such a difficult task. The torsion balance (invented by Charles Coulomb) provides a means of negating the otherwise overwhelming effects of the Earth's attraction in this experiment. It also provides a force delicate enough to counterbalance the tiny gravitational force that exists between the large and small masses. This force is provided by twisting a very thin beryllium copper ribbon.

The large masses are first arranged in Position I, as shown in Figure 12, and the balance is allowed to come to equilibrium. The swivel support that holds the large masses is then rotated, so the large masses are moved to Position II, forcing the system into disequilibrium. The resulting oscillatory rotation of the system is then observed by watching the movement of the light spot on the scale, as the light beam is deflected by the mirror.

Any of three methods can be used to determine the gravitational constant,  $G$ , from the motion of the small masses. In Method I, the final deflection method, the motion is allowed to come to resting equilibrium — a process that requires several hours — and the result is accurate to within approximately 5%. In method II, the equilibrium method, the experiment takes 90 minutes or more and produces an accuracy of approximately 5% when graphical analysis is used in the procedure. In Method III, the acceleration method, the motion is observed for only 5 minutes, and the result is accurate to within approximately 15%.

► **Note:** 5% accuracy is possible in Method I if the experiment is set up on a sturdy table in an isolated location where it will not be disturbed by vibration or air movement.

► **Note:** 5% accuracy is possible in Method II if the resting equilibrium points are determined using a graphical analysis program.

### **METHOD I: Measurement by Final Deflection**

Setup Time: ~ 45 minutes; Experiment Time: several hours

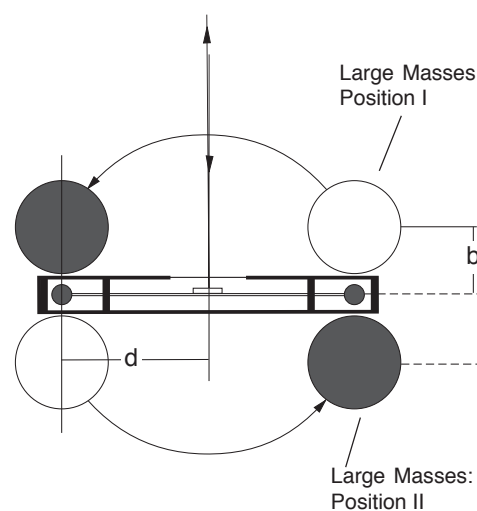
Accuracy: ~ 5%

#### **Theory**

With the large masses in Position I (Figure 13), the gravitational attraction,  $F$ , between each small mass ( $m_2$ ) and its neighboring large mass ( $m_1$ ) is given by the law of universal gravitation:

$$F = Gm_1m_2/b^2 \quad (1.1)$$

where  $b$  = the distance between the centers of the two masses.



**Figure 13**  
Origin of variables  $b$  and  $d$

The gravitational attraction between the two small masses and their neighboring large masses produces a net torque ( $\tau_{grav}$ ) on the system:

$$\tau_{grav} = 2Fd \quad (1.2)$$

where  $d$  is the length of the lever arm of the pendulum bob crosspiece.

Since the system is in equilibrium, the twisted torsion band must be supplying an equal and opposite torque. This torque ( $\tau_{band}$ ) is equal to the torsion constant for the band ( $\kappa$ ) times the angle through which it is twisted ( $\theta$ ), or:

$$\tau_{band} = -\kappa\theta. \quad (1.3)$$

Combining equations 1.1, 1.2, and 1.3, and taking into account that  $\tau_{grav} = -\tau_{band}$ , gives:

$$\kappa\theta = 2dGm_1m_2/b^2$$

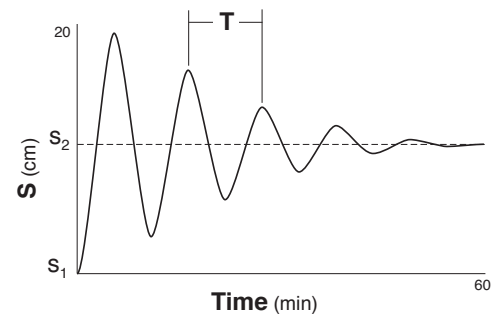
Rearranging this equation gives an expression for  $G$ :

$$G = \frac{\kappa\theta b^2}{2dm_1m_2} \quad (1.4)$$

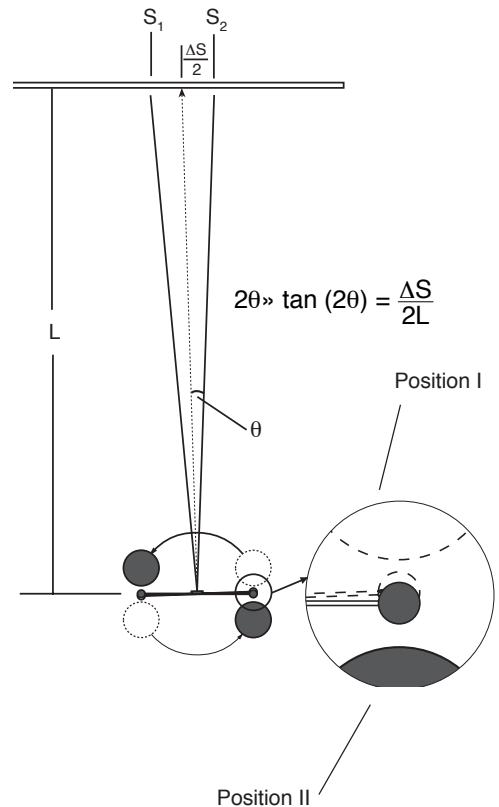
To determine the values of  $\theta$  and  $\kappa$  — the only unknowns in equation 1.4 — it is necessary to observe the oscillations of the small mass system when the equilibrium is disturbed. To disturb the equilibrium (from  $S_1$ ), the swivel support is rotated so the large masses are moved to Position II. The system will then oscillate until it finally slows down and comes to rest at a new equilibrium position ( $S_2$ ) (Figure 14).

At the new equilibrium position  $S_2$ , the torsion wire will still be twisted through an angle  $\theta$ , but in the opposite direction of its twist in Position I, so the total change in angle is equal to  $2\theta$ . Taking into account that the angle is also doubled upon reflection from the mirror (Figure 15):

$$\begin{aligned} \Delta S &= S_2 - S_1, \\ 4\theta &= \Delta S/L \quad \text{or} \\ \theta &= \Delta S/4L \end{aligned} \quad (1.5)$$



**Figure 14**  
Graph of Small Mass Oscillations



**Figure 15**  
Diagram of the experiment showing the optical lever.

The torsion constant can be determined by observing the period ( $T$ ) of the oscillations, and then using the equation:

$$T^2 = 4\pi^2 I / \kappa \quad (1.6)$$

where  $I$  is the moment of inertia of the small mass system.

The moment of inertia for the mirror and support system for the small masses is negligibly small compared to that of the masses themselves, so the total inertia can be expressed as:

$$I = 2m_2(d^2 + 2/5r^2) \quad (1.7)$$

Therefore:

$$\kappa = 8\pi^2 m_2 \frac{d^2 + 2/5r^2}{T^2} \quad (1.8)$$

Substituting equations 1.5 and 1.8 into equation 1.4 gives:

$$G = \pi^2 \Delta S b^2 \frac{(d^2 + 2/5r^2)}{T^2 m_1 L d} \quad (1.9)$$

All the variables on the right side of equation 1.9 are known or measurable:

$$r = 9.55 \text{ mm}$$

$$d = 50 \text{ mm}$$

$$b = 46.5 \text{ mm}$$

$$m_1 = 1.5 \text{ kg}$$

$$L = (\text{Measure as in step 1 of the setup.})$$

By measuring the total deflection of the light spot ( $\Delta S$ ) and the period of oscillation ( $T$ ), the value of  $G$  can therefore be determined.

## Procedure

1. Once the steps for leveling, aligning, and setup have been completed (with the large masses in Position I), allow the pendulum to stop oscillating.
2. Turn on the laser and observe the Position I end point of the balance for several minutes to be sure the system is at equilibrium. Record the Position I end point ( $S_I$ ) as accurately as possible, and indicate any variation over time as part of your margin of error in the measurement.

- Carefully rotate the swivel support so that the large masses are moved to Position II. The spheres should be just touching the case, but take care to avoid knocking the case and disturbing the system.

**Note:** You can reduce the amount of time the pendulum requires to move to equilibrium by moving the large masses in a two-step process: first move the large masses and support to an intermediate position that is in the midpoint of the total arc (Figure 16), and wait until the light beam has moved as far as it will go in the period; then move the sphere across the second half of the arc until the large mass support just touches the case. Use a slow, smooth motion, and avoid hitting the case when moving the mass support.

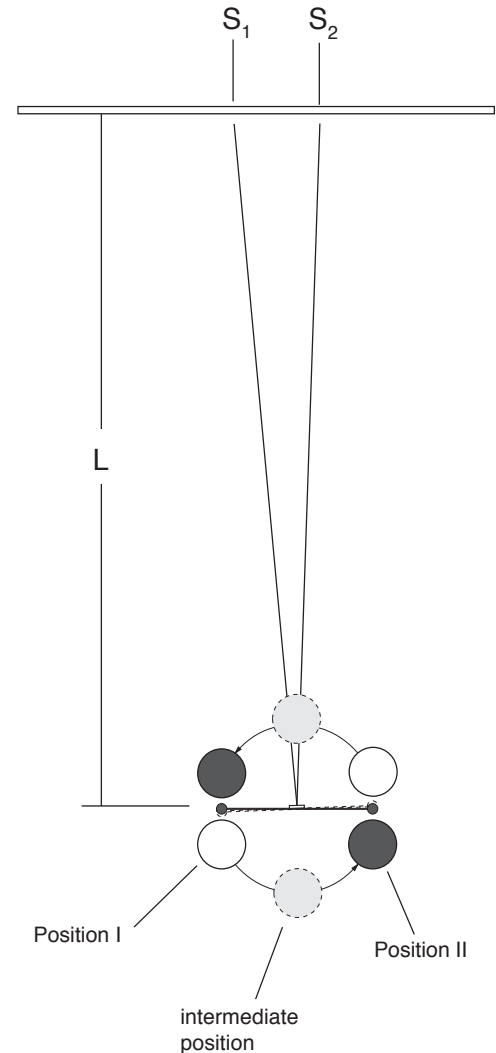
- Immediately after rotating the swivel support, observe the light spot and record its position ( $S_1$ ).
- Use a stop watch to determine the time required for one period of oscillation ( $T$ ). For greater accuracy, include several periods, and then find the average time required for one period of oscillation.

**Note:** The accuracy of this period value ( $T$ ) is very important, since the  $T$  is squared in the calculation of  $G$ .

- Wait until the oscillations stop, and record the resting equilibrium point ( $S_2$ ).

## Analysis

- Use your results and equation 1.9 to determine the value of  $G$ .
- The value calculated in step 2 is subject to the following systematic error. The small sphere is attracted not only to its neighboring large sphere, but also to the more distant large sphere, though with a much smaller force. The



**Figure 16**  
Two-step process of moving the large masses to reduce the time required to stop oscillating

geometry for this second force is shown in Figure 17 (the vector arrows shown are not proportional to the actual forces).

From Figure 17,

$$f = F_0 \sin\Phi$$

$$\sin\Phi = \frac{b}{(b^2 + 4d^2)^{1/2}}$$

The force,  $F_0$  is given by the gravitational law, which translates, in this case, to:

$$F_0 = \frac{Gm_2m_1}{(b^2 + 4d^2)}$$

and has a component  $f$  that is opposite to the direction of the force  $F$ :

$$f = \frac{Gm_2m_1b}{(b^2 + 4d^2)(b^2 + 4d^2)^{1/2}} = \beta F$$

This equation defines a dimensionless parameter,  $b$ , that is equal to the ratio of the magnitude of  $f$  to that of  $F$ . Using the equation  $F = Gm_1m_2/b^2$ , it can be determined that:

$$b = b^3/(b^2 + 4d^2)^{3/2}$$

From Figure 17,

$$F_{net} = F - f = F - bF = F(1 - b)$$

where  $F_{net}$  is the value of the force acting on each small sphere from *both* large masses, and  $F$  is the force of attraction to the nearest large mass only.

Similarly,

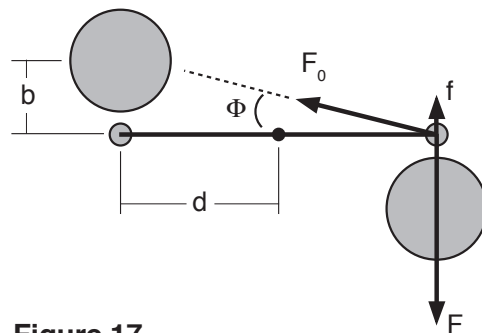
$$G = G_0(1 - b)$$

where  $G$  is your experimentally determined value for the gravitational constant, and  $G_0$  is corrected to account for the systematic error.

Finally,

$$G_0 = G/(1 - b)$$

Use this equation with equation 1.9 to adjust your measured value.



**Figure 17**  
Correcting the measured value of  $G$



## METHOD II: Measurement by Equilibrium Positions

Observation Time: ~ 90+ minutes

Accuracy: ~ 5 %

### Theory

When the large masses are placed on the swivel support and moved to either Position I or Position II, the torsion balance oscillates for a time before coming to rest at a new equilibrium position. This oscillation can be described by a damped sine wave with an offset, where the value of the offset represents the equilibrium point for the balance. By finding the equilibrium point for both Position I and Position II and taking the difference, the value of  $\Delta S$  can be obtained. The remainder of the theory is identical to that described in Method I.

**Note:** To obtain an accuracy of 5% with this method, it is important to use graphical analysis of the position and time data to extrapolate the resting equilibrium positions,  $S_1$  and  $S_2$ .

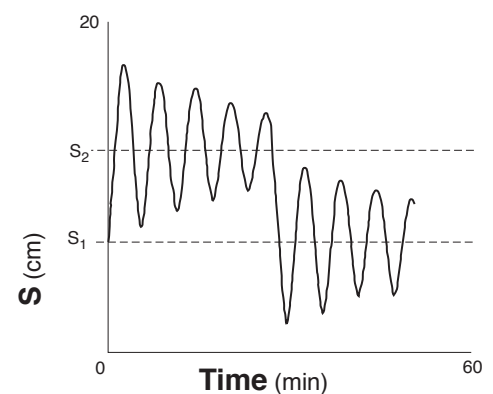
### Procedure

1. Set up the experiment following steps 1–3 of Method I.
2. Immediately after rotating the swivel support to Position II, observe the light spot. Record the position of the light spot ( $S$ ) and the time ( $t$ ) every 15 seconds. Continue recording the position and time for about 45 minutes.
3. Rotate the swivel support to Position I. Repeat the procedure described in step 2.

**Note:** Although it is not imperative that step 3 be performed immediately after step 2, it is a good idea to proceed with it as soon as possible in order to minimize the risk that the system will be disturbed between the two measurements. Waiting more than a day to perform step 3 is not advised.

### Analysis

1. Construct a graph of light spot position versus time for both Position I and Position II. You will now have a graph similar to Figure 18.
2. Find the equilibrium point for each configuration by analyzing the corresponding graphs using graphical analysis to extrapolate the resting equilibrium points  $S_1$  and  $S_2$  (the equilibrium point will be the center line about which the oscillation occurs). Find the difference between



**Figure 18**  
Typical pendulum oscillation pattern showing equilibrium positions

the two equilibrium positions and record the result as  $\Delta S$ .

3. Determine the period of the oscillations of the small mass system by analyzing the two graphs. Each graph will produce a slightly different result. Average these results and record the answer as  $T$ .
4. Use your results and equation 1.9 to determine the value of  $G$ .
5. The value calculated in step 4 is subject to the same systematic error as described in Method I. Perform the correction procedure described in that section (*Analysis, step 3*) to find the value of  $G_0$ .

### **METHOD III: Measurement by Acceleration**

Observation Time:  $\sim 5$  minutes

Accuracy:  $\sim 15\%$

#### **Theory**

With the large masses in Position I, the gravitational attraction,  $F$ , between each small mass ( $m_2$ ) and its neighboring large mass ( $m_1$ ) is given by the law of universal gravitation:

$$F = Gm_1m_2/b^2 \quad (3.1)$$

This force is balanced by a torque from the twisted torsion ribbon, so that the system is in equilibrium. The angle of twist,  $\theta$ , is measured by noting the position of the light spot where the reflected beam strikes the scale. This position is carefully noted, and then the large masses are moved to Position II. The position change of the large masses disturbs the equilibrium of the system, which will now oscillate until friction slows it down to a new equilibrium position.

Since the period of oscillation of the small masses is long (approximately 10 minutes), they do not move significantly when the large masses are first moved from Position I to Position II. Because of the symmetry of the setup, the large masses exert the same gravitational force on the small masses as they did in Position I, but now in the opposite direction. Since the equilibrating force from the torsion band has not

changed, the total force ( $F_{total}$ ) that is now acting to accelerate the small masses is equal to twice the original gravitational force from the large masses, or:

$$F_{total} = 2F = 2Gm_1m_2/b^2 \quad (3.2)$$

Each small mass is therefore accelerated toward its neighboring large mass, with an initial acceleration ( $a_0$ ) that is expressed in the equation:

$$m_2a_0 = 2Gm_1 m_2/b^2 \quad (3.3)$$

Of course, as the small masses begin to move, the torsion ribbon becomes more and more relaxed so that the force decreases and their acceleration is reduced. If the system is observed over a relatively long period of time, as in Method I, it will be seen to oscillate. If, however, the acceleration of the small masses can be measured before the torque from the torsion ribbon changes appreciably, equation 3.3 can be used to determine  $G$ . Given the nature of the motion—damped harmonic—the initial acceleration is constant to within about 5% in the first one tenth of an oscillation. Reasonably good results can therefore be obtained if the acceleration is measured in the first minute after rearranging the large masses, and the following relationship is used:

$$G = b^2a_0/2m_1 \quad (3.4)$$

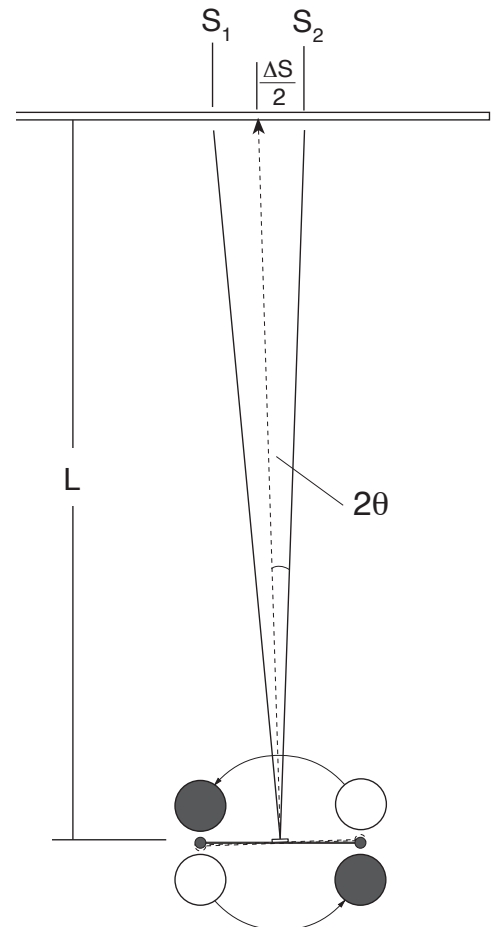
The acceleration is measured by observing the displacement of the light spot on the screen. If, as is shown in Figure 19:

- $\Delta s$  = the linear displacement of the small masses,
- $d$  = the distance from the center of mass of the small masses to the axis of rotation of the torsion balance,
- $\Delta S$  = the displacement of the light spot on the screen, and
- $L$  = the distance of the scale from the mirror of the balance,

then, taking into account the doubling of the angle on reflection,

$$\Delta S = \Delta s(2L/d) \quad (3.5)$$

Using the equation of motion for an object with a constant



**Figure 19**  
Source of data for calculations in Method III

acceleration ( $x = 1/2 at^2$ ), the acceleration can be calculated:

$$a_0 = 2\Delta s/t^2 = \Delta Sd/t^2L \quad (3.6)$$

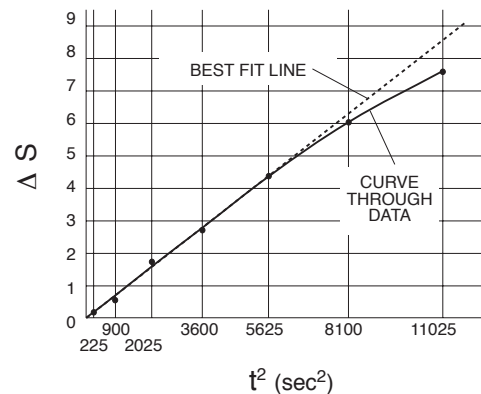
By monitoring the motion of the light spot over time, the acceleration can be determined using equation 3.6, and the gravitational constant can then be determined using equation 3.4.

## Procedure

1. Begin the experiment by completing steps 1–3 of the procedure detailed in Method I.
2. Immediately after rotating the swivel support, observe the light spot. Record the position of the light spot ( $S$ ) and the time ( $t$ ) every 15 seconds for about two minutes.

## Analysis

1. Construct a graph of light spot displacement ( $\Delta S = S - S_1$ ) versus time squared ( $t^2$ ), with  $t^2$  on the horizontal axis (Figure 20). Draw a best-fit line through the observed data points over the first minute of observation.
2. Determine the slope of your best-fit line.
3. Use equations 3.4 and 3.6 to determine the gravitational constant.
4. The value calculated in step 3 is subject to a systematic error. The small sphere is attracted not only to its neighboring large sphere, but also to the more distant large sphere, although with a much smaller force. Use the procedure detailed in Method I (*Analysis, step 3*) to correct for this force.



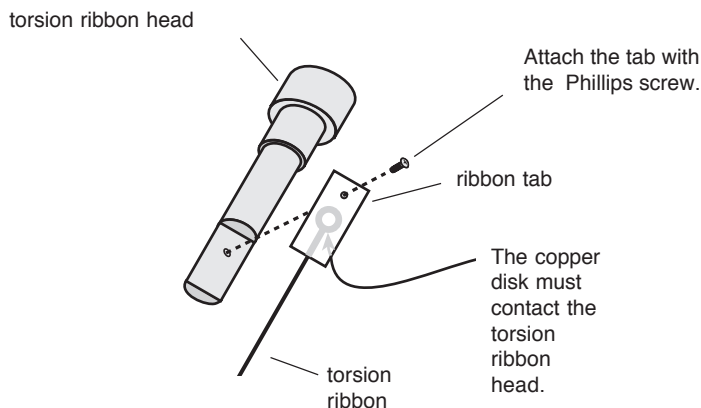
**Figure 20**  
Sample data and best-fit line

# Maintenance

## Replacing the Torsion Ribbon Assembly

If the torsion ribbon breaks, replace it as follows:

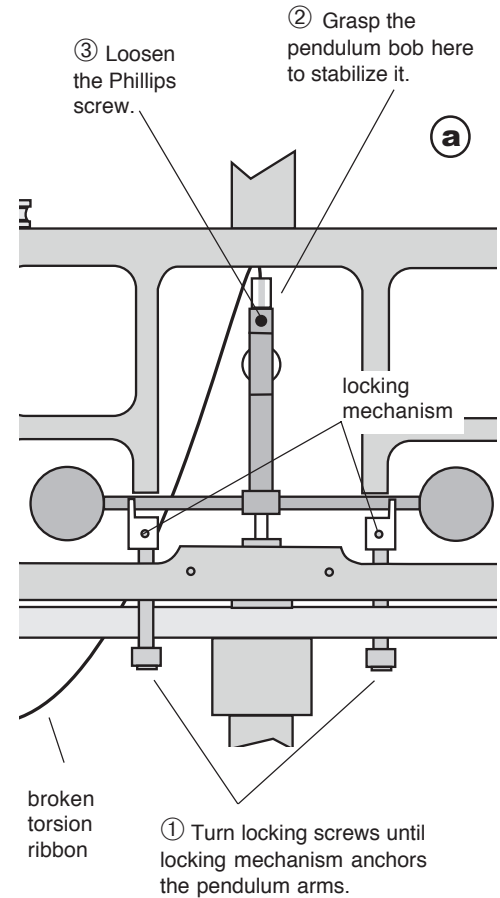
1. Remove the plates, and raise the locking mechanism using the locking screws until the pendulum arms are securely anchored (Figure 21a).
2. Grasp the pendulum bob near the bottom ribbon tab to stabilize it.
3. Loosen the Phillips screw on the bottom tab of the torsion ribbon assembly (Figure 21a), and remove the bottom half of the broken ribbon assembly.
4. Loosen the Phillips screw at the top of the balance assembly (Figure 21b).
5. Grasp the torsion ribbon head and remove the top portion of the broken torsion ribbon assembly.
6. Attach the top tab of the new torsion ribbon to the torsion ribbon head using the Phillips screw, being sure the copper disc on the tab is in contact with the torsion ribbon head (Figure 22). Align the tab with the face of the torsion ribbon head.



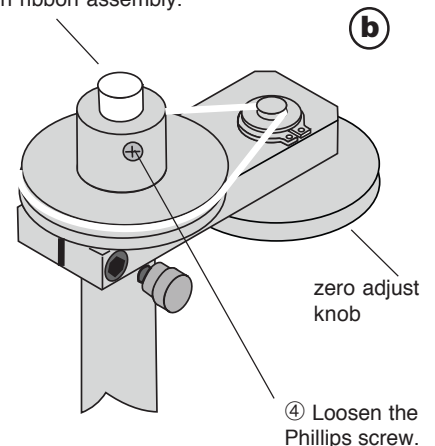
**Figure 22**

Attaching the top tab of the torsion ribbon assembly to the torsion ribbon head

7. Thread the ribbon through the shaft.
8. Using the zero adjust knob, align the bottom tab with the face of the pendulum bob.



5. Grasp the torsion ribbon head and remove the top portion of the broken ribbon assembly.



**Figure 21**

Securing the pendulum bob before removing a broken torsion ribbon, and loosening the torsion ribbon head

 **Note: Be sure the ribbon is not twisted.**

9. Tighten the Phillips screw on the top of the balance to secure the torsion ribbon head.
10. Attach the bottom tab of the ribbon to the pendulum bob using the Phillips screw.
11. Replace the back plate.
12. Level and align the pendulum according to the instructions in the *Equipment Setup* section of this manual.



**Note: To avoid breaking the torsion ribbon, the locking mechanism must be fully raised on both sides during any moving or transporting of the Gravitational Torsion Balance.**

## ***Transporting and Storing***

1. To prepare the Gravitational Torsion Balance for transporting or storing:
  - a. Remove the front plate.
  - b. Raise the locking mechanism to securely anchor the pendulum bob.
  - c. Check to be sure that the torsion ribbon is hanging straight down the center of the tube. If it is not, lower the locking mechanisms, be sure the torsion wire is centered, and raise the locking mechanisms again. Repeat as necessary until the ribbon is centered in the tube.
  - d. Reinstall the packing foam into the chamber to secure the pendulum bob.
  - e. Replace the plate.
2. The Gravitational Torsion Balance may be stored flat in its shipping container.
3. Store in a cool, dry place, and protect the device from any jarring or rough handling.

## ***Safety Precaution***

The small and large masses are made of lead, which is toxic if ingested. Use appropriate precautions for handling lead, including hand washing after handling the masses.

# Technical Support

## Feedback

If you have any comments about the product or manual, please let us know. If you have any suggestions on alternate experiments or find a problem in the manual, please tell us. PASCO appreciates any customer feedback. Your input helps us evaluate and improve our product.

## To Reach PASCO

For technical support, call us at 1-800-772-8700 (toll-free within the U.S.) or (916) 786-3800.

fax: (916) 786-3292

e-mail: [techsupp@pasco.com](mailto:techsupp@pasco.com)

web: [www.pasco.com](http://www.pasco.com)

## Contacting Technical Support

Before you call the PASCO Technical Support staff, it would be helpful to prepare the following information:

► If your problem is with the PASCO apparatus, note:

- Title and model number (usually listed on the label);
- Approximate age of apparatus;
- A detailed description of the problem/sequence of events (in case you can't call PASCO right away, you won't lose valuable data);
- If possible, have the apparatus within reach when calling to facilitate description of individual parts.

► If your problem relates to the instruction manual, note:

- Part number and revision (listed by month and year on the front cover);
- Have the manual at hand to discuss your questions.

